Signal Processing -Exercices

1 Exercice 1

Let $x(t) = a\Pi_T(t)$ where $\Pi_T(t)$ is the rectangular function defined as $\Pi_T(t) = 1$ for all $t \in [-T/2, T/2]$ and is null everywhere else.

1. Calculate the deterministic correlation function $Rxx(\tau) = \int_{-\infty}^{+\infty} x(t)x^*(t-\tau)$.
2. Calculate the Fourier transform of y(t)=x*x(t). (Hint : Y(f)=a$^2 T^2 sinc(\pi f T)^2$).

2 Exercice 2

Let $x(t)$ be a real continuous signal. Let $z(t)$ be defined as $Z(f) = 2U(f)X(f)$, where $U(f)$ is the frequency domain Heaviside function such that $U(f) = 0$ if $f < 0$, $U(f) = 1$ for $f > 0$ and $U(0) = 0.5$.

1. Let us first consider $p(t) = \mathrm{Re}(z(t)) = (z(t)+z^*(t))/2$. Prove that $P(f) = X(f)$. Conclude that $\mathrm{Re}(z(t)) = x(t)$.
2. Let us now consider $q(t) = \mathrm{Im}(z(t)) = (z(t) - z^*(t))/(2j)$. Prove that $Q(f) = -jsign(f)X(f)$. Hint : $sign(f) = U(f) - U(-f)$. What could you say ?
3. Conclude that $z(t) = x(t) + j\hat{x}(t)$ where $\hat{x}(t)$ is defined as the filtered version of $x(t)$ with the filter $h(t)$ with complex gain $H(f) = -jsign(f)$. $\hat{x}(t)$ is referred to as the Hilbert transform of $x(t)$ whereas $z(t)$ is the so-called analytic signal.

3 Exercice 3

Let $x(t)$ be a real continuous signal, band-limited to B Hz (X(f) has support on [-B/2,B/2]). This signal is filtered by $h(t)$ defined in the Fourier domain as $H(f) = (1 + kcos(2\pi fT))e^{-j2\pi f t_d}$ for all $|f| < B$, and null everywhere else.

1. Compute the Fourier transform of $y(t) = h * x(t)$.
2. Deduce $y(t)$ and conclude that $y(t)$ is a dispersed version of $x(t)$ due to some echoes.

4 Exercice 4

Let $x(t) = Acos(2\pi f_0 t + \phi) + b(t)$ where ϕ is a random variable uniformly distributed in $[0, 2\pi]$. A and f_0 are constants. $b(t)$ is a white noise independent from ϕ. We consider the random process $y(t) = h * x(t)$ where $h(t) = \frac{1}{T}\Pi_T(t - T/2)$.

1. Show that the process $y(t)$ can be seen as the instantaneous average of $x(t)$. Is the system stable ?
2. What is the mean of $y(t)$? Is it a first order stationary process ?
3. Is the process $y(t)$ a second order stationary process ?
4. Compute the Power spectrum density of the random signal $y(t)$.